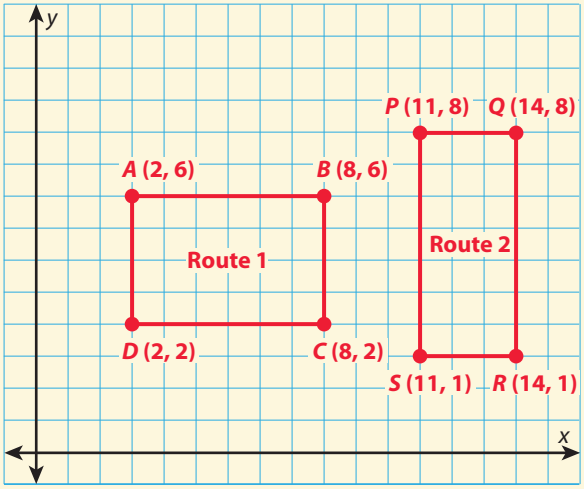
Lesson 23 Introduction

Polygons in the Coordinate Plane

Use What You Know

You've learned about plane figures such as rectangles and triangles. In this lesson you will learn how to use coordinates to analyze plane figures. Take a look at this problem.

Cheryl walks her dogs after school. The routes she takes are shown on the coordinate grid to the right. Route 1 starts at point *A* and continues to points *B*, *C*, *D*, and back to *A*. Route 2 starts at *P* and goes to *Q*, *R*, *S*, and back to *P*. Which route is longer?



Use the math you already know to solve this problem.

a. Which pairs of points on Route 1 have the same *x*-coordinates? _____

_____

The same *y*-coordinates? _____

_____

b. Look at Route 1. Find the distance from *A* to *B*: _____; from *B* to *C*: _____; from *C* to *D*: _____; and from *D* to *A*: _____. The total distance is _____.

c. Look at Route 2. Find the distance from *P* to *Q*: _____; from *Q* to *R*: _____; from *R* to *S*: _____; and from *S* to *P*: _____. The total distance is _____.

d. Explain how you can find which of the two routes is longer.

_____

_____

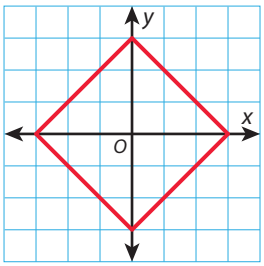
Find Out More

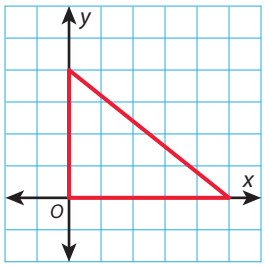
Each of Cheryl's routes on the previous page forms a rectangle on a coordinate plane. A rectangle is one type of polygon. A **polygon** is a closed plane figure whose sides are line segments that intersect only at their endpoints.

On the coordinate plane, if a side of a polygon is on a horizontal line, then its endpoints have the same *y*-coordinate. If a side of a polygon is on a vertical line, its endpoints have the same *x*-coordinate.

One way to find the length of a horizontal or vertical segment is by counting. The length of a segment is always a positive number.

It is often helpful to place a vertex or a side of the polygon on one of the axes of the coordinate plane, as shown in the diagrams below.





Reflect

1 Explain how you could find the distance between points (3, 6) and (7, 6) on the coordinate plane.

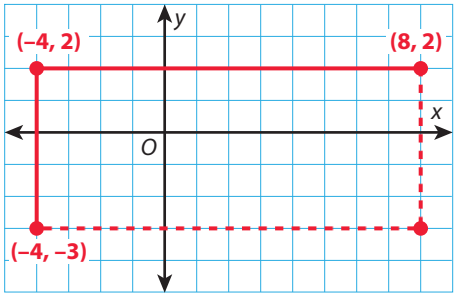
Learn About Finding Area on a Coordinate Plane

Read the problem below. Then use what you know about rectangles and coordinates to find missing coordinates and dimensions.

An athletic director is planning to refinish the floor of a rectangular athletic court. Three corners of the court have the coordinates (−4, 2), (−4, −3), and (8, 2). Find the coordinates of the fourth corner and the perimeter of the court.

Graph It

You can graph the information that is given and then sketch the rectangle.



Draw a horizontal line that goes through (−4, −3) and a vertical line that goes through (8, 2). The point where they intersect is the fourth corner of the court.

To find the perimeter of the court, find its length and width. The length is the distance from (−4, 2) to (8, 2). The width is the distance from (−4, 2) to (−4, −3).

Model It

You can use words to describe the location of the fourth corner of the athletic court.

The fourth corner of the rectangle is on a vertical line that goes through (8, 2), so its *x*-coordinate is 8.

The fourth corner of the rectangle is on a horizontal line that goes through (−4, −3), so its *y*-coordinate is −3.

Connect It **Now you will solve the problems from the previous page using your understanding of coordinates and polygons.**

2 What are the coordinates of the fourth corner of the rectangle? Explain how you found it.

_____

_____

3 Explain how to find the distance between (−4, 2) and (8, 2) using absolute value.

_____

_____

4 Explain how to count to find the distance between (−4, 2) and (−4, −3).

_____

_____

5 Describe how to find the perimeter of the athletic court. _____

_____

_____

_____

6 Explain why it is useful to know how to find distances on the coordinate plane both by counting and by using absolute value. _____

_____

_____

Try It **Use what you just learned about polygons on a coordinate plane to solve these problems. Show your work on a separate sheet of grid paper.**

7 The coordinates of three corners of a square are (−2, 0), (1, 0), and (1, −3). Graph these three points. What are the coordinates of the fourth corner of the square? _____ Draw the square.

8 What is the perimeter of the square in problem 7? _____

Learn About Polygons in the Coordinate Plane

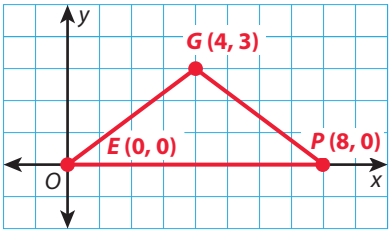
Read the problem below. Then explore how to find the area of a polygon on a coordinate plane.

On a map of a county park, the park entrance is located at (0, 0), a community garden is located at (4, 3), and a playground is located at (8, 0).

- What shape is formed by a path connecting the three locations?
- The park's director is planning to grow grass inside the shape formed by the path. What is the area of the shape?

Graph It

You can make a graph showing the given information to help solve the problem.



The park entrance is labeled *E*, the community garden is labeled *G*, and the playground is labeled *P*.

The polygon is a triangle with a base of 8 units and a height of 3 units.

Solve It

You can use the formula for the area of a triangle to solve the problem.

$$\text{Area} = \left(\frac{1}{2}\right) \times \text{base} \times \text{height}$$

$$A = \left(\frac{1}{2}\right)bh$$

$$= \left(\frac{1}{2}\right)(8)(3)$$

$$= \left(\frac{1}{2}\right) 24$$

$$= 12$$

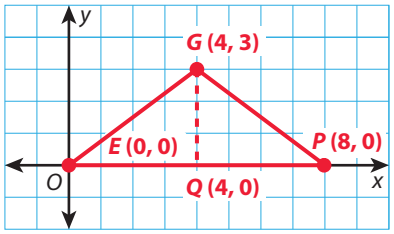
The area of the triangle is 12 square units.

Connect It **Now you will explain how to solve the problem from the previous page using your understanding of polygons on a coordinate plane.**

9 Explain how you know that the base of the triangle is 8 units long.

10 Explain how you know that the height of the triangle is 3 units long.

11 Kristina divided Δ*EGP* into two smaller triangles by drawing a vertical line from *G* to the *x*-axis, as shown in the diagram.



What do you notice about Δ*EGQ* and Δ*PGQ*?

12 How could you use Kristina's method to find the area of Δ*EGP*?

13 Suppose that the coordinates of point *G* were (4, −3) instead of (4, 3). Would the area of Δ*EGP* be the same? Explain why or why not.

Try It **Use what you just learned about finding the area of a polygon on the coordinate plane to solve these problems. Show your work on a separate sheet of paper.**

A baseball diamond is in the shape of a square, with bases at (0, 4), (4, 0), (0, −4), and (−4, 0). The pitcher's mound is located at (0, 0).

14 What is the area of the part of the square that is in Quadrant I?

15 What is the total area of the square?

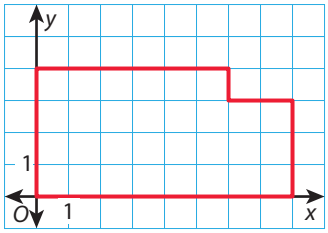
Practice Drawing and Analyzing Polygons in the Coordinate Plane

Study the example below. Then solve problems 16–18.

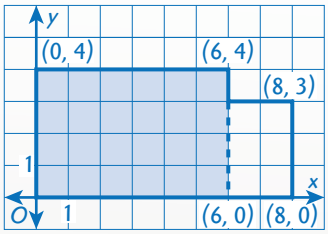
The student divided the polygon into two rectangles and found the area of each one.

Example

Find the area of the polygon shown below.



Look at how you can divide the polygon into shapes whose areas are easy to find.



Area of shaded rectangle: $(6)(4) = 24$

Area of unshaded rectangle: $(2)(3) = 6$

Solution The total area is $24 + 6 = 30$ square units.

Pair/Share
Is there another way to divide the shape into two rectangles?

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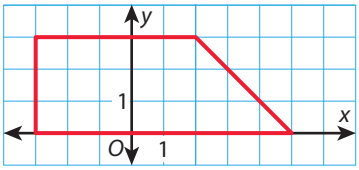
16 Find the area of the trapezoid.



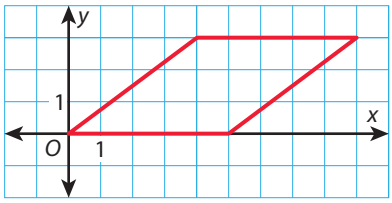
How can you separate this figure into different shapes?

Show your work.

Pair/Share
Explain the steps you used to find the area.

Solution _____

17 Find the area of the parallelogram.



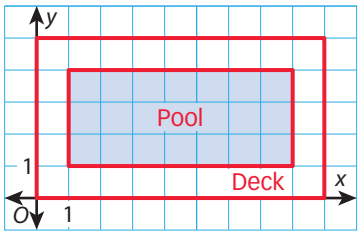
Show your work.

Solution _____

What is the formula for a parallelogram's area?

Pair/Share
Could you find this area by dividing the parallelogram into smaller polygons?

18 A swimming pool and the deck surrounding it are shown in the coordinate plane below. What is the area of the deck? Circle the letter of the correct answer.



A 66 square units

B 21 square units

C 24 square units

D 45 square units

Ron chose **D** as his answer. Why is this answer incorrect?

_____

_____

_____

_____

_____

What are the dimensions of each of the rectangles in the diagram?

Pair/Share
Explain what the correct answer should be.

Practice Drawing and Analyzing Polygons in the Coordinate Plane

Solve the problems.

1 Three corners of a rectangular city block are located at (2, 2), (2, −4), and (−5, −4) on a coordinate plane. What are the coordinates of the fourth corner?

A (−5, 8)

B (5, −2)

C (−5, 2)

D (2, −5)

2 What is the perimeter of the rectangular city block in problem 1?

A 13 units

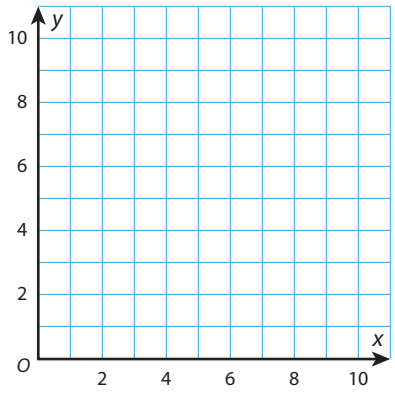
B 26 units

C 42 units

D 10 units

3 On the coordinate plane, plot the following points. Then, connect the points in order from *A* to *E* and then back to *A* to form a figure. What is the area of the figure?

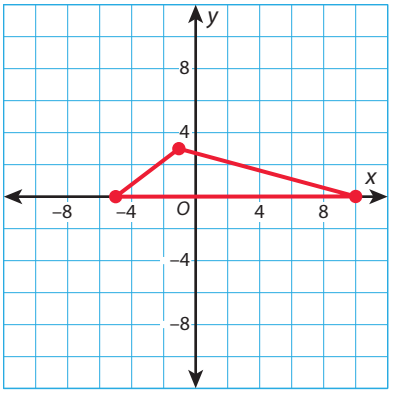
A(1, 2) *B*(1, 7) *C*(9, 7) *D*(9, 3) *E*(6, 5)



The area of the figure is _____ square units.

4 Which triangle has an area of 22.5 square units? Circle all that apply.

A



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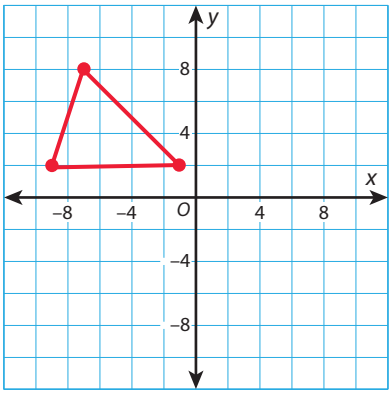
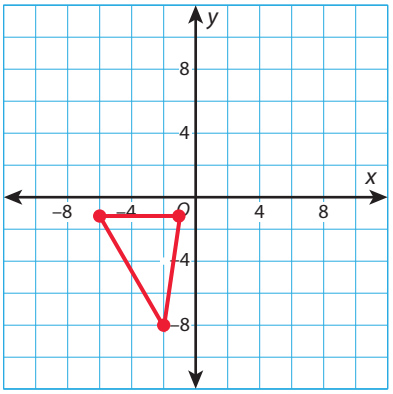
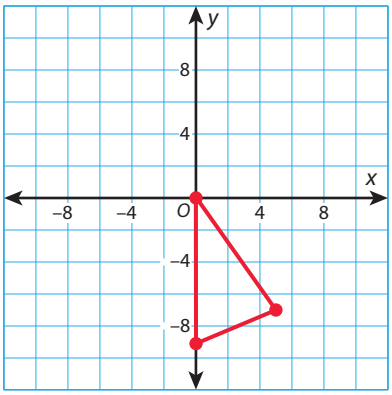


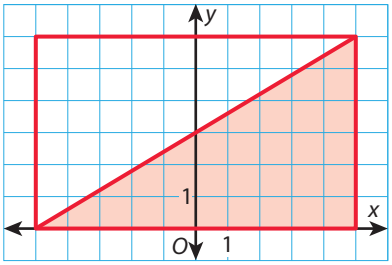
B



D



5 The rectangle on the coordinate plane at the right is a diagram of a flag. Each of the large triangles on the flag will be a different color. Is the same amount of material needed for each color? Explain why or why not.



6 Two vertices of a triangle are (0, 0) and (8, 0). The *x*-coordinate of the third vertex is 4. Imagine that you draw, on a coordinate plane, several triangles that meet all these conditions. What would be the same about all the triangles you drew? What would be different?

Self Check **Go back and see what you can check off on the Self Check on page 225.**